Physics 105

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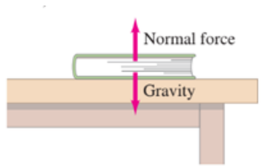
ACADEMY

Chapter -9-
(Static equilibrium, Elasticity and fracture)

❖ Section (9.1): The conditional for equilibrium

- Equilibrium: A state of rest or balance due to the equal action of opposing forces.
  - ➢ For **example**,consider a book at rest on a table. Does this mean that the book is not affected by forces? Of course not. The book is affected by the force of gravity acting downward and the normal force exerted by the table in the opposite direction.
  - ➢ The book remains at rest because the net force acting on it is zero. This means that the normal force is equal in magnitude to the force of gravity but acts in the opposite direction. This situation is described as being in equilibrium.



- **Conditions** for static equilibrium:

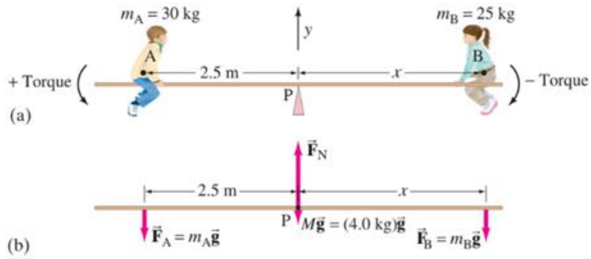
1. $\sum F = 0 \quad \rightarrow \sum F_x = 0 \ , \sum F_y = 0 \ , \sum F_z = 0$ (**No** translational motion)
2. $\sum \tau = 0$ (**No** rotational motion)

   - Note that ***both*** conditions must met together

❖ Section (9.2): Solving statics problems

✓ ***Example:*** A board of mass serves as a seesaw for two children. Child A has a mass of 30 kg and sits 2.5 m from the pivot point, P (his center of gravity is 2.5 m from the pivot). At what distance x from the pivot must child B, of mass 25 kg, place herself to balance the seesaw?
(Assume the board is uniform and centered over the pivot)

✓ ***Solution:***



To get a balance seesaw

1. $\sum F = 0$
2. $\sum \tau = 0$

I. $\sum F_y = F_N - Mg - m_A g - m_B g = 0$

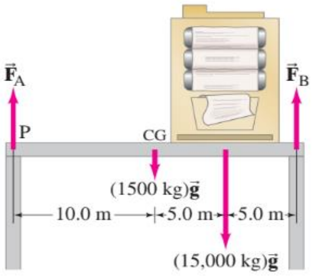
$$F_N = g\ (M + m_A + m_B)$$
$$= g\,(4 + 30 + 25)$$
$$= 578.2N$$

II. $\sum \tau = F_A(2.5) - F_B\,(x) + Mg\,(0) + F_N\,(0) = 0$

$$m_A g\,(2.5) - m_B g\,(x) = 0$$
$$m_A g\,(2.5) = m_B g\,(x)$$
$$\frac{75}{m_B} = \frac{m_B\,(x)}{m_B}$$
$$x = 3m$$

✓ ***Example:*** A uniform 1500-kg beam, 20.0 m long, supports a 15,000-kg printing press 5.0 m from the right support column. Calculate the force on each of the vertical support columns.

✓ ***Solution:***



I. $\sum F_y = 0$

$F_A - Mg - mg + F_B = 0$

$F_A + F_B = g\ (1500 + 15000)$

$F_A + F_B = 161700 \longrightarrow$ **1**

II. $\sum \tau = 0$

F_B with the clockwise direction, so the torque is negative.

$\sum \tau = -F_B\,(20) + Mg\,(10) + mg\ (15) = 0$

$-F_B(20) + 1500 * 9.8 * 10 + 15000 * 9.8 * 15 = 0$

$-20\,F_B + 147000 + 2205000 = 0$

$F_B = 117600\ N$

To find F_A , back to equ. **(1)**

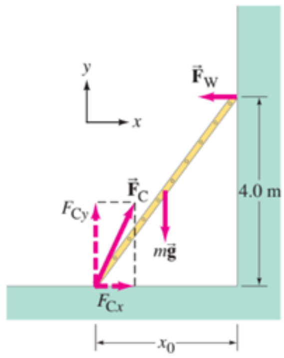
$F_A + F_B = 161700$

$F_A = 161700 - 117600$

$F_A = 44100$ N

✓ ***Example:*** A 5.0-m-long ladder leans against a wall at a point 4.0 m above a cement floor. The ladder is uniform and has mass Assuming the wall is <u>frictionless</u>, but the floor is not, determine the forces exerted on the ladder by the floor and by the wall.

✓ ***Solution:***



1. $\sum F = 0$, In this example we have two dimensions (x, y)

2. $\sum \tau = 0$

$\sum F_x \longrightarrow f_{floor\,x} - F_w = 0$

$\sum F_y \longrightarrow f_{floor\,y} - mg = 0$

$\longrightarrow f_{floor\,y} = mg \qquad f_{floor\,y} = 117.6\ N$

$\sum \tau = 0 \longrightarrow$ for $F_{mg\ ladder}$ the lever arm is half the floor distance.

$x_0 = \sqrt{(5)^2 + (4)^2} = 3m \longrightarrow f_{mg\ ladder} \longrightarrow r = 1.5\ m$

$\sum \tau \longrightarrow F_w(4) - (1.5)mg = 0$

$F_w = 44.1\ N$

$f_{floor\,x} - F_w = 0$

Then $f_{floor\,x} = F_w = 44.1\ N$

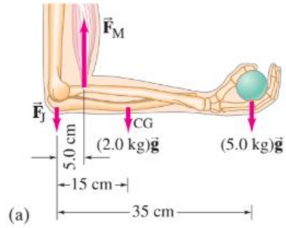
$f_{floor} = \sqrt{(f_{floor\,x})^2 + (f_{floor\,y})^2} = 125.596\ N$

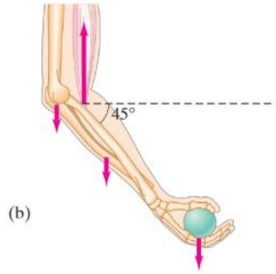
$\theta = tan^{-1}\left|\frac{f_{floor\,y}}{f_{floor\,x}}\right| \longrightarrow \theta = 69.44^{\circ}$

❖ Section (9.3): Applications to Muscles and Joints

- ✓ ***Example:*** How much force must the biceps muscle exert when a 5.0-kg ball is held in the hand (a) with the arm horizontal, and (b) when the arm is at a 45° angle. The biceps muscle is connected to the forearm by a tendon attached 5.0 cm from the elbow joint. Assume that the mass of fore arm and hand together is 2.0 kg and their CG is as shown.

- ✓ ***Solution:***





1. $\sum F = 0$

2. $\sum \tau = 0$

(a) $\sum \tau = 0$

$(0.05)F_M - (0.15)F_{CG} - (0.35)F_{ball} = 0$

$(0.05)F_M - (0.15)(2)g - (0.35)\ (5)g = 0$

$F_M = 401.8\ N$

$\sum F_y = 0$

$F_M - F_j - F_{CG} - F_{ball} = 0$

$4018 - F_j - F_{CG} - F_{ball} = 0$

$F_j = 333.2\ N$

(b) $\sum \tau = 0$

$(0.05)\ F_M\ \cancel{\cos 45^\circ} - (0.15)\ F_{CG}\ \cancel{\cos 45^\circ} - (0.35)\ F_{ball}\ \cancel{\cos 45^\circ} = 0$

$FM\ (0.05) - (0.15)\ 2\ g - (0.35)\ 5\ g = 0$

$FM = 401.8\ N$

❖ Section (9.4): Stability and Balance

- An object in static equilibrium, if left undisturbed, will undergo no translational or rotational acceleration since the sum of all the forces and the sum of all the torques acting on it are zero. However, if the object is displaced slightly, three outcomes are possible:
  (1) The object returns to its original position, in which case it is said to be in *stable equilibrium*
  (2) The object moves even farther from its original position, and it is said to be in *unstable equilibrium*
  (3) The object remains in its new position, and it is said to be in *neutral equilibrium*.
- In general, an object whose center of gravity (CG) is below its point of support such as a ballon on a string will be in a ***stable equilibrium***.
- Also we can say that any object whose center of gravity is above its base of support **will be stable**.

❖ Section (9.5): Elasticity, stress and strain

- In this section, we will examine how forces affect an object, including changes in its length, shape, and the impact of applied forces. We will also explore whether the object will break if the forces are strong enough.

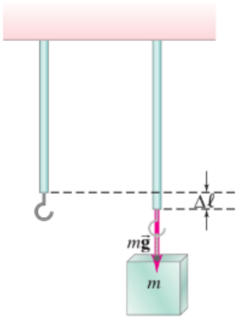
- **Elasticity:** The ability of ***deformed material*** body **to return on its original shape** and **size** when the forces causing the **deformation are moved**.

- **Hooke's Law:**

$$F = K\ \Delta l$$

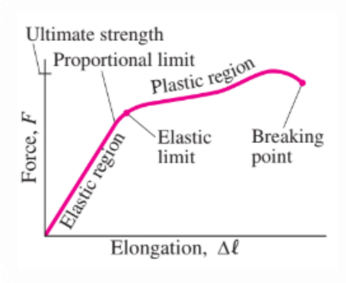
Where:

  - F is the applied force.
  - K is the proportional constant.
  - Δl is the change of length under the effect of the force.



- The above relation is almost _valid_ for any material from iron to born.

  - ➢ Elastic region: ***Hook's law applies***. $F = K\ \Delta l$
    and ***object returns*** to the original length when the force removed, where is limited from the origin to the *elastic limit*.
  - ➢ Elastic limit: ***Maximum value*** of Δl at which the object will return to its original length once the ***force is removed***.
  - ➢ Breaking point: ***The maximum force*** that can be applied without the object breaking.
  - ➢ Plastic region: Region from *elastic limit* to breaking point, In this region the object becomes ***permanently deformed***.



- There are *three types* of deformation:

  I. Young's Modules: measure the **resistance** of solid to *change in its length*

$$\Delta l = \frac{1}{E}\frac{F}{A}\ l_\circ$$

TABLE 9–1 Elastic Moduli

Material	Young's Modulus, E (N/m²)	Shear Modulus, G (N/m²)	Bulk Modulus, B (N/m²)
Solids			
Iron, cast	100×10^9	40×10^9	90×10^9
Steel	200×10^9	80×10^9	140×10^9
Brass	100×10^9	35×10^9	80×10^9
Aluminum	70×10^9	25×10^9	70×10^9
Concrete	20×10^9		
Brick	14×10^9		
Marble	50×10^9		70×10^9
Granite	45×10^9		45×10^9
Wood (pine) (parallel to grain)	10×10^9		
(perpendicular to grain)	1×10^9		
Nylon	$\approx 3 \times 10^9$		
Bone (limb)	15×10^9	80×10^9	
Liquids			
Water			2.0×10^9
Alcohol (ethyl)			1.0×10^9
Mercury			2.5×10^9
Gases[†]			
Air, H_2, He, CO_2			1.01×10^5

[†]At normal atmospheric pressure; no variation in temperature during process.

  - E is a proportionality constant known as Young's modulus. The value of E **depends** on the material type but is **independent** of the material's shape or size. Its *unit* is **N/m².**

✓ ***Example***: A 1.6m long steel piano wire has a diameter of 0.2cm. How great is the tension in the wire if it stretches 0.25cm when tightened? ($E\ \ for\ steel\ = \ 200 * \ 10^9\ N/\ m^2$)

✓ ***Solution:***

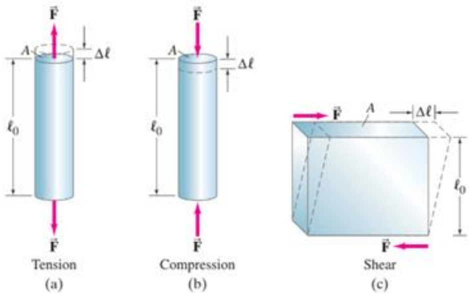
$$\Delta l = \frac{1}{E}\frac{F}{A}\ l_\circ$$

$$A\ for\ wire = \pi\, r^2\ = 3.14 * \left(\frac{0.2}{2}\right)^2\ = 0.0314\ cm^2\ = \ 3.14 * \ 10^{-6}\ m^2$$

$$F = E\ \frac{\Delta l}{l_\circ}\ A$$

$$F\ = 200 * \ 10^9 * \frac{0.25 * 10^{-2}}{1.6} * 3.14 * \ 10^{-6}\ = \ 981.25\ \text{N}$$

- The three types of **stress** for rigid objects:
  1. Tension
  2. Compression
  3. Shear



II. Shear Modules: measures the **resistance** to motion of the planes within **a solid parallel** to each other. (*the dimensions* of the object *doesn't change* much, but the *shape changes*)

- Stress: Force per unit area **F/A**. has units of N/m²
- Strain: Ratio of change in length to original length $\frac{\Delta l}{l_\circ}$

$$E = \frac{F}{A} * \frac{l_\circ}{\Delta l} = \frac{stress}{strain}$$

$$\therefore \ \text{Strain} = \frac{\text{Stress}}{\text{E}}$$

- In shear modulus we can write

$$\Delta \boldsymbol{l} = \frac{\mathbf{1}}{\boldsymbol{G}} \frac{\boldsymbol{F}}{\boldsymbol{A}} \ \boldsymbol{l}_\circ$$

- Where G is a constant of proportionality is called **shear modulus**. It is generally one half to one third the value of young's modulus. Its unit is **N/m²**.

$$\text{Shear strain} = \frac{1}{\text{G}} \text{shear stress}$$

- The force acts could be tensile stress, compressive stress and shear stress.

Δl *larger* when the object is *thicker* and its original length ($l_\circ$)is *greater*.

III. Bulk modulus: measures the **resistance** of solids or fluids to *changes in their volume*.

- The force acts in all directions ⟶ pressure. In which $\boldsymbol{P} = \frac{\boldsymbol{F}}{\boldsymbol{A}}$ is ***equivalent*** to stress.
- In Bulk modulus we can write

$$\frac{\Delta \boldsymbol{V}}{\boldsymbol{V}_\circ} = -\frac{\mathbf{1}}{\boldsymbol{B}} \ \Delta \boldsymbol{P}$$

Where:

- $V_\circ$ is the original volume.
- ΔV is the change in volume due to pressure (stress).
- B is a constant proportionality called bulk modulus.

- Bulk modulus: (The Minus sign means the volume decreases with an increase pressure)

$$\boldsymbol{B} = \frac{-\Delta \boldsymbol{P} \ (\boldsymbol{V}_\circ)}{\Delta \boldsymbol{V}}$$

✓ ***Example:*** A student carries a 21-kg bag in one hand. Assuming the humerus (the upper arm bone) supports the entire weight of the bag, determine the amount by which the bone stretches. (The humerus may be assumed to be 33cm in length and to have an effective cross-sectional area of 5.2 x $10^{-4}\ m^2$ when E = 15 × $10^9\ \ N/\ m^2$).

✓ ***Solution:***

$\Delta\, l = \frac{1}{E}\,\frac{F}{A}\, l_\circ$ only F_{mg} acts on the bag then $F_{mg} = mg\ 205.8\ N$

$$\Delta\, l = \frac{1}{15 * 10^9} * \frac{205.8}{5.2 * 10^{-4}} * 0.33m \longrightarrow \Delta l = 8.70 * 10^{-6}\ m$$

✓ ***Example:*** A solid brass sphere is initially surrounded by air, and the air pressure exerted on it is 1.0 * 10^5 N/m^2 , the normal atmospheric pressure. The sphere is lowered into the ocean to a depth where the pressure is 2.0 *10^7 N/m^2. The volume of the sphere in air is 0.50 m^3. By how much does this volume change once the sphere is submerged?

✓ ***Solution:***

$$\Delta\, V = -\frac{1}{B}\,\Delta\, P\, V_\circ$$

$$= -\frac{1}{180 * 10^9} * (2 * 10^7 - 1 * 10^5) * 0.5$$

$$= -1.24 * 10^{-4}\ m^3$$

❖ Section (9.6): Fracture

- **A fracture** occurs when the stress on an object ***<u>becomes so great</u>*** that the object **<u>breaks</u>**

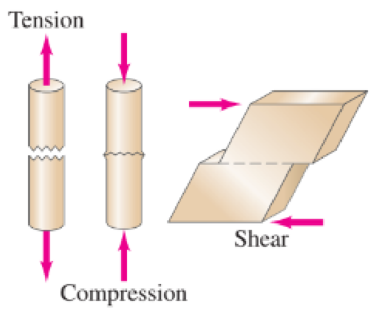


TABLE 9–2 Ultimate Strengths of Materials (force/area)

Material	Tensile Strength (N/m^2)	Compressive Strength (N/m^2)	Shear Strength (N/m^2)
Iron, cast	170×10^6	550×10^6	170×10^6
Steel	500×10^6	500×10^6	250×10^6
Brass	250×10^6	250×10^6	200×10^6
Aluminum	200×10^6	200×10^6	200×10^6
Concrete	2×10^6	20×10^6	2×10^6
Brick		35×10^6	
Marble		80×10^6	
Granite		170×10^6	
Wood (pine) (parallel to grain)	40×10^6	35×10^6	5×10^6
(perpendicular to grain)		10×10^6	
Nylon	500×10^6		
Bone (limb)	130×10^6	170×10^6	

✓ ***Example:*** The steel piano wire was 1.60 m long with a diameter of 0.20 cm. Approximately what tension force would break it? ($\frac{F}{A}$ for steel $= 500 * 10^6\ \ N/m^2$)

✓ ***Solution:***

$$\frac{F}{A} = 500 * 10^6\ \ N/m^2$$

$$A_{for\ wire} = \pi\, r^2 = 3.14 * \left(\frac{0.2}{2}\right)^2 = 0.0314\ cm^2 = 3.14 * 10^{-6}\ m^2$$

Then F = A × 500 × 10^6 = $3.14 * 10^{-6} * 500 * 10^6 = 1570\ N$

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